Spacecraft Attitude Control Using Nonlinear Servomechanism Theory*

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Abstract: The recently developed nonlinear servomechanism theory is applied to the attitude control of large maneuvering spacecraft subject to sinusoidal disturbances and parametric uncertainties. Simulation shows significant advantages of our attitude controller over the controller resulting from the feedback linearization approach.

1. Introduction

Future spacecraft present specific and difficult control problems, largely due to their nonlinear dynamics and uncertainties. To better account for nonlinearity and uncertainties inherent in the spacecraft dynamics, many researchers have applied various nonlinear control approaches to meet these challenges. The typical nonlinear approaches include input-output feedback linearization and sliding mode control [SI], [SL]. However, it is well known that the feedback linearization technique is less capable of dealing with the external disturbance acting on the torques and spacecraft parameter variations. The sliding mode control, though effective in counteracting system nonlinearities and uncertain ies, often incurs excessive control power due to the chattering phenomenon,

The recently developed nonlinear servomechanism (or alternatively, output regulation) theory [HR1], [IB], [HR2], and [HL] provides a promising tool to design control system for spacecraft subject to persistent disturbances. Roughly, the servomechanism theory aims to design a controller for a plant such that the output of the plant asymptotically tracks a reference input and rejects a disturbance. Both the disturbance and reference are generated by an autonomous differential equation called the exosystem. In contrast to the earlier developed inversion-based approaches such as sliding mode control, and input-output feedback linearization the nonlinear servomechanism theory is based only on the existence of steady state inverted d ynamics of the given system, It can handle, therefore, a larger class of nonlinear systems, e.g., non-minimum phase nonlinear systems, Other advantages of this approach include that it can more easily accommodate disturbances, and that it *offers* a nonlinear version of the "*separation principle*" that leads to an effective means to synthesize an output feedback control law based on a state feedback control law and a nonlinear observer.

In this paper, we will formulate the spacecraft attitude control problem aa a nonlinear tracking problem subject to sinusoidal disturbances and parametric uncertainties. In Section 2, we summarize the basic results of the nonlinear servomechanism theory. Section 3 designs a spacecraft attitude controller using the nonlinear servomechanism theory. Section 4 presents some simulation results on the performance of this controller along with a comparison to the the feedback linearization controller. Finally, some remarks are provided in Section 5.

2. Nonlinear Servomechanism Problem

Consider the plant described by

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t), w(t)),\ 2'(0) = x_0 \\ y(t) &= /l(z(t), U(t),\ w(t)),\ t \geq 0 \end{aligned} \qquad (2.1)$$

where $x(t)$ is the n-dimensional plant state, $u(t)$ *is* the m-dimensional plant input, $y(t)$ is the p-dimensional plant output, and $w(t)$ is the q_1-dimensional disturbance signal. The disturbance signal is assumed to be generated by the q-dimensional exogenous system

$$\dot{w}(t) = a_1(w(t)),\ w(0) = w_0 \qquad (2.2)$$

where w_0 may be known or unknown.

In addition, there is a reference input generated by a q_2-dimensional exogenous system

$$\dot{r}(t) = a_2(r(t)),\ r(0) = r_0 \qquad (2.3)$$

where r. is assumed to be known, and the tracking error is defined by

$$e(t) = y(t) - d(r(t)) \qquad (2.4)$$

For simplicity, all the functions involved in this setup are assumed to be smooth and defined globally on the appropriate Euclidean spaces, with the value zero at the respective origins. All our results are stated locally in terms of open neighborhoods of origins of appropriate Euclidean spaces.

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We will consider the nonlinear feedback control law of the general form

$$\begin{aligned} u(t) &= k(x(t), z(t), r(t), w(t)) \\ \dot{z}(t) &= g(x(t), z(t), y(t), r(t)) \end{aligned} \tag{2.5}$$

where $z(t)$ is the compensator state of dimension n_c to be specified later, and the functions $k(\cdot,\cdot,\cdot,\cdot)$ and $g(\cdot,\cdot,\cdot,\cdot)$ are required to be smooth and zero for zero arguments. With an abuse of notation, the control law (2,5) encompasses two most interesting cases:
(i) State feedback when $n_c = 0$, that is,

$$u(t) = k(x(t), r(t), w(t)) \tag{2,6}$$

(ii) Output dynamic feedback when $x(t)$ and $w(t)$ do not explicitly appear in Equation (2.5), that is,

$$\begin{aligned} u(t) &= k(z(t), r(t)) \\ \dot{z}(t) &= g(z(t), y(t), r(t)) \end{aligned} \tag{2.7}$$

To formulate the requirements on the closed-loop system, let $x_c(t) = (x(t)\ z(t))$; then, the closed-loop system can be described by

$$\begin{aligned} \dot{x}_c(t) &= f_c(x_c(t), r(t), w(t)), \quad x_c(0) = x_{c0} \\ y(t) &= h_c(x_c(t), r(t), w(t)), \quad t \geq 0 \end{aligned} \tag{2.8}$$

where $h_c(\cdot,\cdot,\cdot)$ and $f_c(\cdot,\cdot,\cdot)$ are defined as

$$\begin{aligned} h_c(x_c, r, w) &= h(x, k(x, z, r, w), w) \\ f_c(x_c, r, w) &= \begin{bmatrix} f(x, k(x, z, r, w), w) \\ g(x, z, h_c(x_c, r, w), r) \end{bmatrix} \end{aligned} \tag{2.9}$$

The basic requirements for the closed-loop system can be described as follows. First, the equilibrium of $f_c(x, 0, 0)$ at the origin is asymptotically stable, which can always be guaranteed by placing the eigenvalues of the matrix

$$\frac{\partial f_c}{\partial x}(0,0,0) \tag{2.10}$$

in the left half-plane. The second requirement is that, for all sufficient] y small initial conditions x_{c0}, r_o, and w_o, the tracking error (2.4) satisfies

$$\lim_{t \to \infty} e(t) = 0 \tag{2.11}$$

If there exists a control law of the form *(2.5)* such that the closed-loop system satisfies the above two requirements, we say that the nonlinear servomechanism problem is (locally) solvable. Alternatively, we say that the control law achieves asymptotic tracking and disturbance rejection in the plant. In the sequel, such a control law is called a **servo-regulator.**

Before stating the basic result of the nonlinear servomechanism theory, we make the following standard assumptions [IB]:

A1: $\{\frac{\partial f}{\partial x}(0,0,0), \frac{\partial f}{\partial u}(0,0,0)\}$ is stabilizable.

A2: The pair

$$\left[\frac{\partial h}{\partial x}(0,0,0) \quad \frac{\partial h}{\partial w}(0,0,0) \right] \quad \begin{bmatrix} \frac{\partial f}{\partial x}(0,0,0) & \frac{\partial f}{\partial w}(0,0,0) \\ 0 & \frac{\partial a_1}{\partial w}(0) \end{bmatrix}$$

is detectable.
A3: The equilibrium of ecosystems (2.2) and (2.3) is stable, and all the eigenvalues of $\frac{\partial a_1}{\partial w}(0)$ and $\frac{\partial a_2}{\partial r}(0)$ have zero real parts.

Theorem 2.1: Under assumptions A1 and A3, suppose there exist two smooth functions $\mathbf{x}(r, w)$, $\mathbf{u}(r, w)$ defined in a neighborhood of the origin such that $\mathbf{x}(0, 0) = 0$, $\mathbf{u}(0, 0) = 0$, and

$$\begin{aligned} \frac{\partial \mathbf{x}(r,w)}{\partial w} a_1(w) + \frac{\partial \mathbf{x}(r,w)}{\partial r} a_2(r) &= f(\mathbf{x}(r,w), \mathbf{u}(r,w), w) \\ h(\mathbf{x}(r,w), \mathbf{u}(r,w), w) - d(r) &= 0 \end{aligned} \tag{2.12}$$

Then, there exist a state feedback control law of the form (2.6) such that the closed-loop system satisfies the two requirements (2.10) and (2.11).

If the assumptions in Theorem 2.1 are satisfied, then a desired state feedback control law can be formulated as follows,

$$u(t) = \mathbf{u}(r(t), w(t)) + K(x(t) - \mathbf{x}(r(t), w(t))) \tag{2,13}$$

where the feedback gain matrix K *is* such that all the eigenvalues of the matrix

$$\frac{\partial f}{\partial x}(0,0,0) + \frac{\partial f}{\partial u}(0,0,0) K \tag{2.14}$$

have negative real parts, Clearly, the existence of the matrix K is guaranteed by the assumption A1. The solvability conditions for (2.12) can be established using the center manifold theory [C] and are discussed in [IB], [HL] in detail.

Remark 2.1: It is interesting to note that, given $r(t)$ and $w(t)$, the trajectories of the closed-loop system starting from $x_0 = \mathbf{x}(r(0), w(0))$ under control input $u(t) = \mathbf{u}(r(t), w(t))$ is given by $\mathbf{x}(r(t), w(t))$, and the tracking error is identically zero for all $t \geq 0$. For this reason, the functions $\mathbf{x}(r, w)$ and $\mathbf{u}(r, w)$ can be viewed as the steady-state response and steady-state control input required for maintaining zero steady-state tracking error. From a geometric point of view, the set $\{\mathbf{x}(r, w), r, w\}$ can be viewed as a zero-error invariant manifold with respect to the control $\mathbf{u}(r, w)$. The role of the feedback control (2.13) is to render the manifold certain stability properties such that the trajectories starting from points sufficiently close to the manifold asymptotically approach the manifold.

Remark 2.2: The control law (2,13) consists of two parts, namely, a feedback gain K that stabilizes the closed-loop system, and two feedforward functions $\mathbf{x}(r, w)$ and $\mathbf{u}(r, w)$ that annihilate the steady-state tracking error. Note that while the feedback gain can be readily obtained via a variety of methods such as eigenvalue assignment, H^2, and H^∞, the feed forward functions have to be solved from the partial differential equation (2.12). Note that, as shown in [IB] and [HL], solving (2.12) may be formidable for general nonlinear systems, yet is straightforward for input-ouput feedback linearizable nonlinear systems, such as the spacecraft system to be introduced in Section 3.

Remark 2.3: In the case where the plant state z and/or the disturbance state w are unavailable, it is possible to synthesize, under the additional assumption A2, an output-feedback control law:

$$\begin{aligned} u(t) &= k(z(t), r(t)) \\ \dot{z}(t) &= g(z(t), y(t), r(t)) \end{aligned} \tag{2.15}$$

where

$$k(z,r) = \mathbf{u}(r, z_2) + K[z_1 - x(r, z_2)]$$

$$g(z,y,r) = \begin{bmatrix} f(z_1, k(z,r), z_2) \\ a_1(z_2) \end{bmatrix} + G\,[y - h(z_1, k(z,r), z_2)]$$

with $z_1 \in R^{n}$, $z_2 \in R^{q_1}$, and $z = (z_1, z_2)$, and G is such that all the eigenvalues of the matrix

$$\begin{bmatrix} \frac{\partial f}{\partial x}(0,0,0) & \frac{\partial f}{\partial w}(0,0,0) \\ 0 & \frac{\partial a_1}{\partial w}(0) \end{bmatrix} + G \begin{bmatrix} \frac{\partial h}{\partial x}(0,0,0) & \frac{\partial h}{\partial w}(0,0,0) \end{bmatrix}$$

have negative real parts.

3. Application to Spacecraft Attitude Control

We consider the attitude control problem for a spacecraft in a circular orbit in an inverse square gravitational field, and assume that the attitude of the space vehicle has no effect on the orbit. This problem has been investigated in [SI] using the sliding mode control method. Following the treatment in [SI], we can derive the spacecraft dynamics as follows. Let $\hat{x}_1, \hat{x}_2, \hat{x}_3$ be the frame of principle axes of the spacecraft, referred to as the spacecraft frame in the sequel. Let ξ_1, ξ_2, ξ_3 be the reference frame defined as follows: ξ_3 is along the local radius vector from the gravitational center E_1 and through the spacecraft center of mass E; ξ_2 is normal to the plane of orbit; and ξ_1, ξ_2, ξ_3 forms a dextral right-handed system.

Kinematic Equations

Let $\omega = (\omega_1, \omega_2, \omega_3)^T$ denote the angular velocity vector of the spacecraft, expressed in the spacecraft frame. Let $\theta = (\theta_1, \theta_2, \theta_3)^T$ be the pitch, yaw, and roll angles, respectively. Then, the equations describing the evolution of the spacecraft angular velocity with respect to the spacecraft frame is given by

$$\begin{aligned} \omega &= \begin{bmatrix} (\omega_0 + \dot{\theta}_1)sin\theta_2 + \dot{\theta}_3 \\ (\omega_0 + \dot{\theta}_1)cos\theta_2 cos\theta_3 + \dot{\theta}_2 sin\theta_3 \\ -(\omega_0 + \dot{\theta}_1)cos\theta_2 sin\theta_3 + \dot{\theta}_2 cos\theta_3 \end{bmatrix} \\ &\stackrel{def}{=} R(\theta)\dot{\theta} + \omega_c(\theta) \end{aligned} \tag{3.1}$$

where ω_0 denotes the constant orbital angular velocity of the mass center of the spacecraft; and

$$R(e) = \begin{bmatrix} sin\theta_2 & 0 & 1 \\ cos\theta_2 cos\theta_3 & sin\theta_3 & 0 \\ -cos\theta_2 sin\theta_3 & cos_3 & 0 \end{bmatrix} \tag{3.2}$$

$$\omega_c(\theta) = \begin{bmatrix} \omega_0 sin\theta_2 \\ \omega_0 cos\theta_2 cos\theta_3 \\ -\omega_0 cos\theta_2 sin\theta_3 \end{bmatrix} \tag{3.3}$$

Solving (3.1) for $\dot{\theta}$ gives the kinematic equations as follows

$$\dot{\theta} = R^{-1}(\theta)(\omega - \omega_c(\theta)) \tag{3.4}$$

where

$$R^{-1}(\theta) = \begin{bmatrix} 0 & cos\theta_3 sec\theta_2 & -sin\theta_3 sec\theta_2 \\ 0 & sin\theta_3 & cos\theta_3 \\ 1 & -tan\,\theta_2 cos\theta_3 & tan\,\theta_2 sin\theta_3 \end{bmatrix} \tag{3.5}$$

Equation (3.5) is valid in the region $-\pi/2 < \theta_2 < \pi/2$. (other representations use 4 "Euler parameters" to avoid such singularities)

Dynamic Equations

The dynamical equation of motion about the center of mass of the spacecraft is

$$I\dot{\omega} + \tilde{\omega} I \omega = 3\omega_0^2 \tilde{\xi}_c(\theta) I \xi_c(\theta) + u + T_d \tag{3.6}$$

where $I = diag(I_1, I_2, I_3)$ is the moment of inertia of the spacecraft with respect to the spacecraft frame; $u = (u_1, u_2, u_3)^T$ is the control torque vector acting about the axes $\hat{x}_1, \hat{x}_2, \hat{x}_3$; $T_d = (T_{d1}, T_{d2}, T_{d3})^T$ is a sinusoidal disturbance torque vector of the form

$$T_{di} = T_i sin(\omega_{di} t),\ i = 1,2,3 \tag{3.7}$$

with ω_{di} fixed and T_i arbitrary; the vector $\xi_c(\theta)$ is

$$\xi_c(\theta) = \begin{bmatrix} \xi_{c1}(\theta) \\ \xi_{c2}(\theta) \\ \xi_{c3}(\theta) \end{bmatrix} = \begin{bmatrix} -sin\theta_1 cos\theta_2 \\ cos\theta_1 sin\theta_3 + sin\theta_1 sin\theta_2 cos\theta_3 \\ cos\theta_1 cos\theta_3 - sin\theta_1 sin\theta_2 sin\theta_3 \end{bmatrix}$$

and $\tilde{\omega}$ and $\tilde{\xi}_c$ are skewsymmetric matrices given by

$$\tilde{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix},\quad \tilde{\xi}_c = \begin{bmatrix} 0 & -\xi_{3c} & \xi_{2c} \\ \xi_{3c} & 0 & -\xi_{1c} \\ -\xi_{2c} & \xi_{1c} & 0 \end{bmatrix}$$

Let $x = (\theta^T, \omega^T)^T$. Then, combining (3.4) and (3.6) gives the following state space equations

$$\begin{aligned} \dot{x} &= f(x, T_d) + g(x)u \\ y &= h(x) \end{aligned} \tag{3.8}$$

where

$$\begin{aligned} f(x, T_d) &= \begin{bmatrix} R^{-1}(\theta)(\omega - \omega_c(\theta)) \\ I^{-1}(-\tilde{\omega} I \omega + 3\omega_0^2 \tilde{\xi}_c(\theta) I \xi_c(\theta) + T_d) \end{bmatrix} \\ g(x) &= \begin{bmatrix} 0_{3\times 3} \\ I^{-1} \end{bmatrix}, \quad h(x) = \theta \end{aligned} \tag{3.9}$$

The attitude control problem is described as follows: derive a control law u such that in the closed-loop system $\theta(t)$ asymptotically tracks a reference trajectory $\theta_r(t) = (\theta_{r1}(t), \theta_{r2}(t), \theta_{r3}(t))^T$ in the presence of the parametric uncertainties and external disturbances, Here we assume the reference trajectories take the form

$$\theta_{ri}(t) = C_i(1 - exp(-\alpha_i t))\,(deg)\; i = 1,2,3$$

with C_i and α_i constant,

To address the above attitude control problem using the nonlinear servomechanism theory, we first define the two ecosystems as follows:

$$\dot{w} = A_w w,\; w(0) = w_0 \tag{3.10}$$

where

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix}, \quad w_0 = \begin{bmatrix} 0 \\ T_1 \\ 0 \\ T_2 \\ 0 \\ T_3 \end{bmatrix}$$

$$A_w = \begin{vmatrix} 0 & \omega_{d1} & 0 & 0 & 0 & 0 \\ -\omega_{d1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_{d2} & 0 & 0 \\ 0 & 0 & -\omega_{d2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_{d3} \\ 0 & 0 & 0 & 0 & -\omega_{d3} & 0 \end{vmatrix}$$

and

$$\dot{r} = A_r r,\; r(0) = r_0 \tag{3.11}$$

where

$$r = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}, \quad r_0 = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}, \quad A_r = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus the disturbance is given by

$$T_d = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} w \overset{def}{=} Fw,$$

As for the reference input, we have $\theta_r = r(1 - exp(-\alpha_i t)) \approx r$. Note that this approximation does not affect the steady state tracking error since $lim_{t\to\infty}\theta_r = r$. Clearly, (3.9)-(3.11) are exactly in the form of (2.1)-(2.3) and satisfy assumptions A1-A3. Thus, the nonlinear servomechanism design method detailed in Section 2 is directly applicable to the attitude controller design for the spacecraft,

Feedback Gain Design: Here we only consider the state feedback case. The feedback gain is designed based on the linearized system around the equilibrium $(\theta, w) = (0, 0)$. The feedback gain is such that the eigenvalues of the linearized closed-loop system are $(-1.414 \pm 1.414i, -1.414 \pm 1.414i, -1.414 \pm 1.414i)$. This feedback gain will approximately decouple the motions of the three axes and provide suitable damping and response speed.

Feedforward Compensator Design: The feedforward compensator is obtained by solving for $x(r, w)$ and $u(r, w)$ from equation (2.12) which takes the following specific expression:

$$\begin{aligned} &\frac{\partial\theta(r,w)}{\partial w}A_w w + \frac{\partial\theta(r,w)}{\partial r}A_r r \\ &= R^{-1}(\theta(r,w))(\omega(r,w) - \omega_c(r)) \qquad \text{(3S2)} \\ &\frac{\partial\omega(r,w)}{\partial w}A_w w + \frac{\partial\omega(r,w)}{\partial r}A_r r \\ &= I^{-1}(-\tilde{\omega}(r,w)I\omega(r,w) + \omega_0^2\tilde{\xi}_c(r)I\xi_c(r) \\ &+ Fw + u) \qquad (3.13) \\ 0 &= \theta(r,w) - r \qquad (3.14) \end{aligned}$$

From equation (3.14), we get

$$\theta(r, w) = r \tag{3.15}$$

Solving for $\omega(r, w)$ from (3.12) (which is in fact an algebraic equation since the right side of (3.12) is equal to zero due to the special forma of $\theta(r, w)$ and A_r) gives

$$w(r, w) = \omega_c(r) \tag{3.16}$$

Finally from (3.13), we obtain

$$u(r, w) = \tilde{\omega}(r,w)I\omega(r,w) - 3\omega_0\tilde{\xi}_c(r)I\tilde{\xi}_c(r) - Fw \tag{3.17}$$

4. Simulation

The performance of the controller is evaluated through computer simulation. The parameters of the spacecraft are taken from [S1] and are given as follows:

$$I_1 = 874.2, I_2 = 888.2, I_3 = 97.7, \omega_0 = 7.29 \times 10^{-5}(rad/s^2)$$

Other parameters are $(T_1, T_2, T_3) = (50,50,50)$, $(\omega_{d1}, \omega_{d2}, \omega_{d3}) = (\pi,\pi,\pi)$, $(C_1, C_2, C_3) = (90,45,30)$, and $(\alpha_1, \alpha_2, \alpha_3) = (1,1,1)$. Two cases are studied. Case 1 is the nominal case, that is, no disturbance is present. In case 2 the spacecraft is subject to both disturbance and a plus 20 % inertial parameter variation, Figures 1 and 2 show the attitude tracking performance for case 1 and case 2, respectively. It is seen that the presence of the disturbance and parametric uncertainties causes little performance degradation. For comparison, we also simulated the performance of the feedback linearization controller designed in [AGNC] for the same cases. We found that the feedback linearization controller acts almost equally well as the nonlinear servo-regulator for case 1. However, for case 2, the performance of the feedback linearization controller great] y deteriorates. Due to limitations of space, here we only present the simulation results for case 2 as shown in Figure 3. The main reason for the good performance of the servo-regulator is that the steady state input (3.17) explicitly incorporates the disturbance state w that cancels the effect of the disturbance, Note that here we only considered the state feedback case. It is possible to extend our design to the output feedback case to eliminate the need of explicitly incorporating the disturbance state w.

5. Concluding Remarks

The nonlinear servomechanism theory has been applied to the attitude control of spacecraft subject to disturbances and parametric uncertainties. Simulations showed the advantage of this controller over the feedback linearization controller. The current design will be extended to the output feedback case.

6. Acknowledgement

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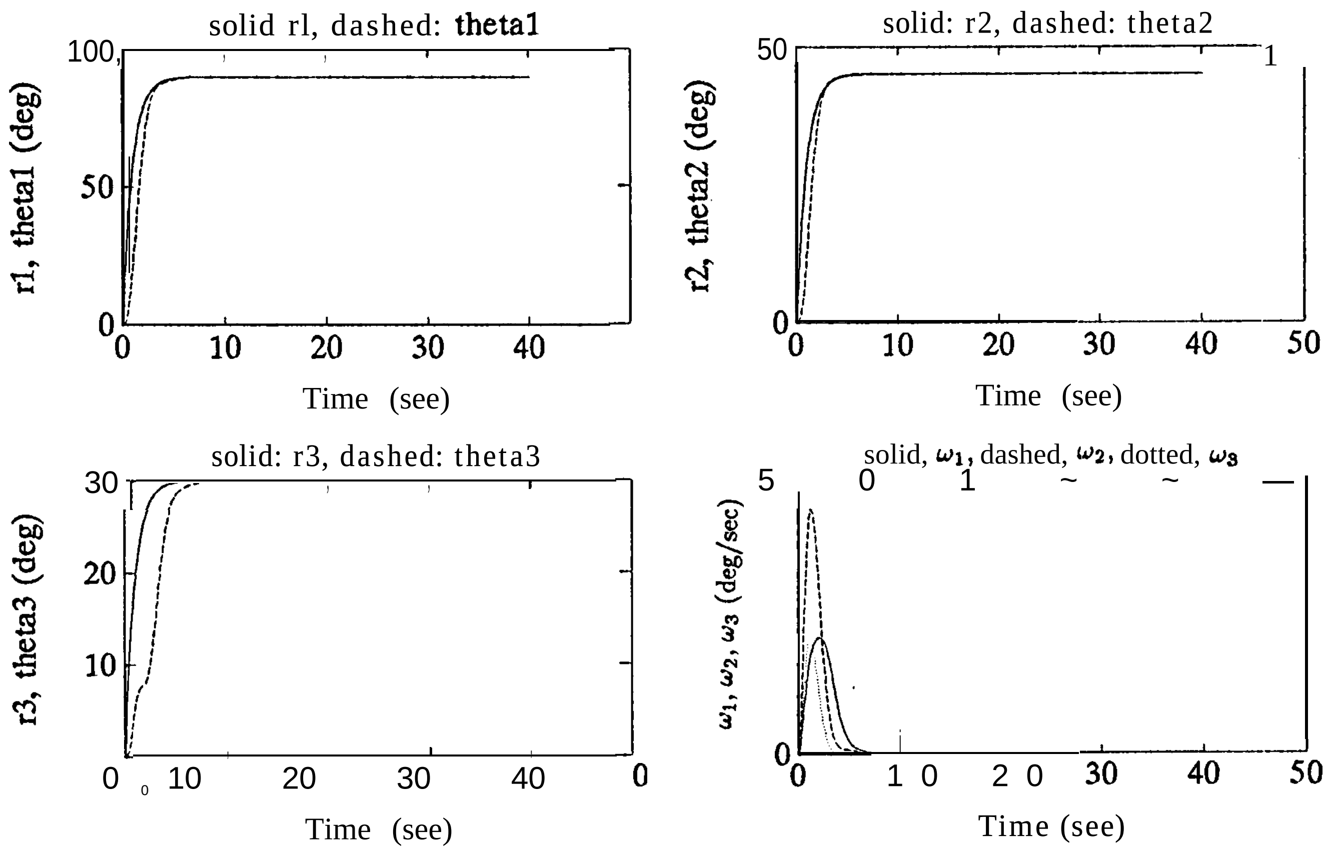

Figure 1: Tracking performance of the nonlinear servo-regulator: Case 1.

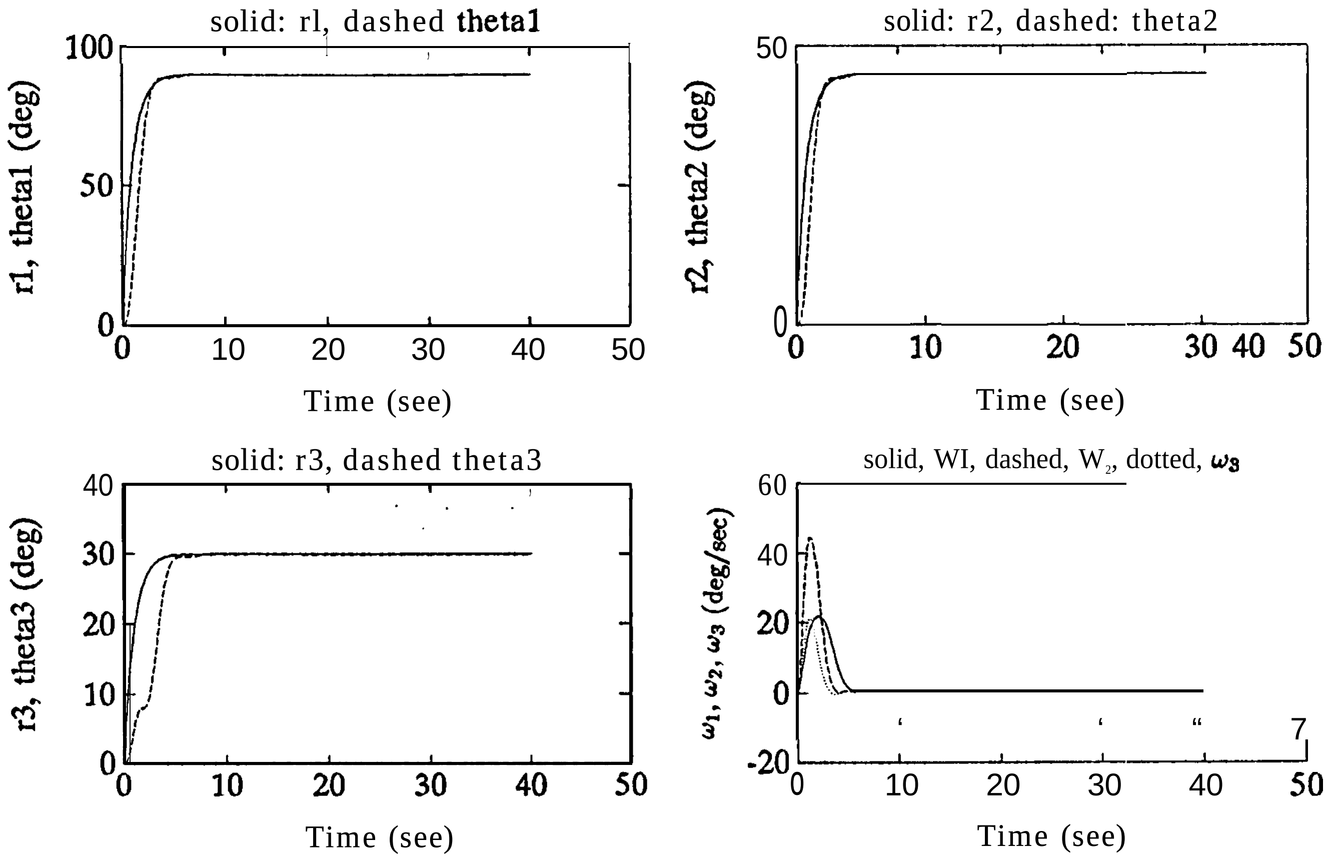

Figure 2: Trading performance of the nonlinear servo-regulator: Case 2.

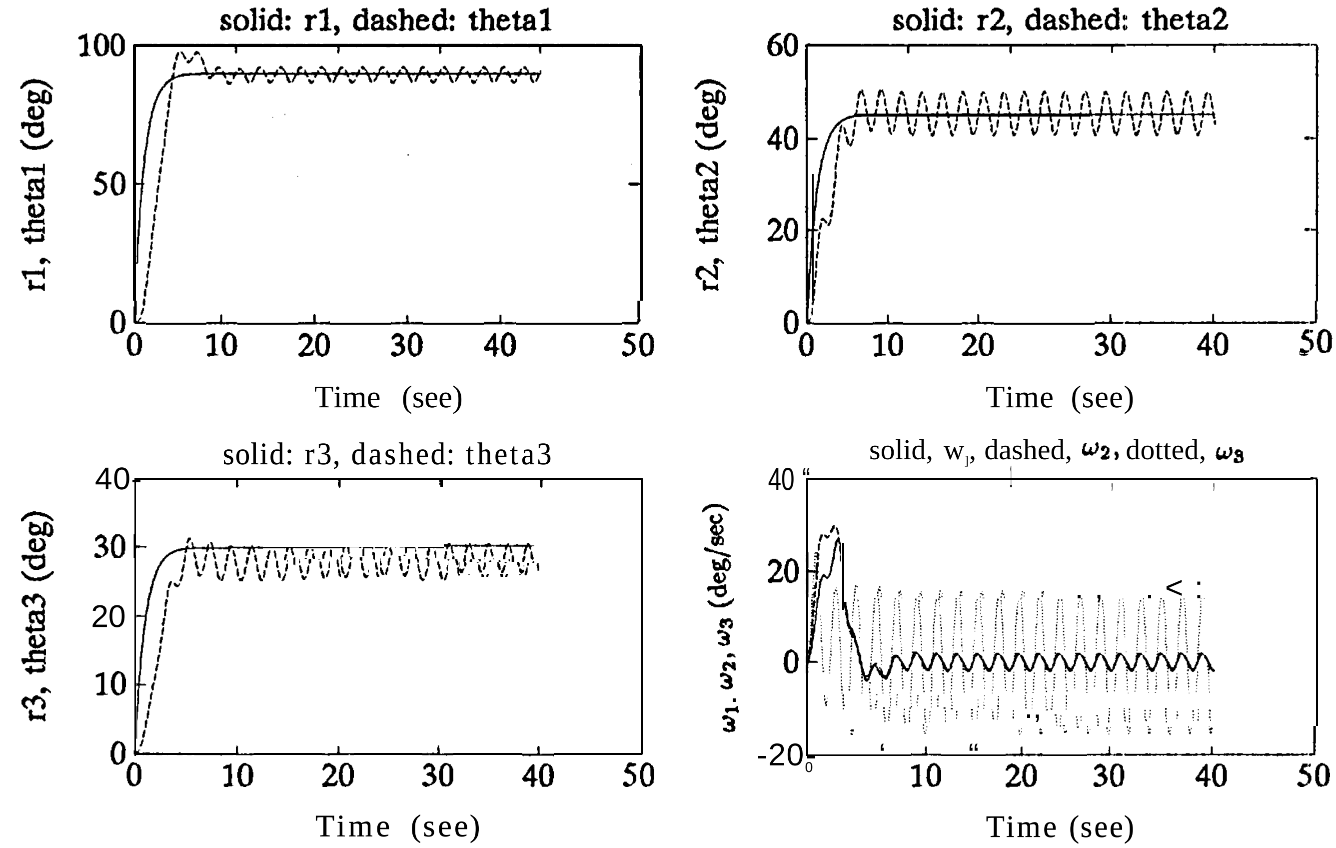

Figure 3: Tracking performance of the feedback linearization controller: Case 2.